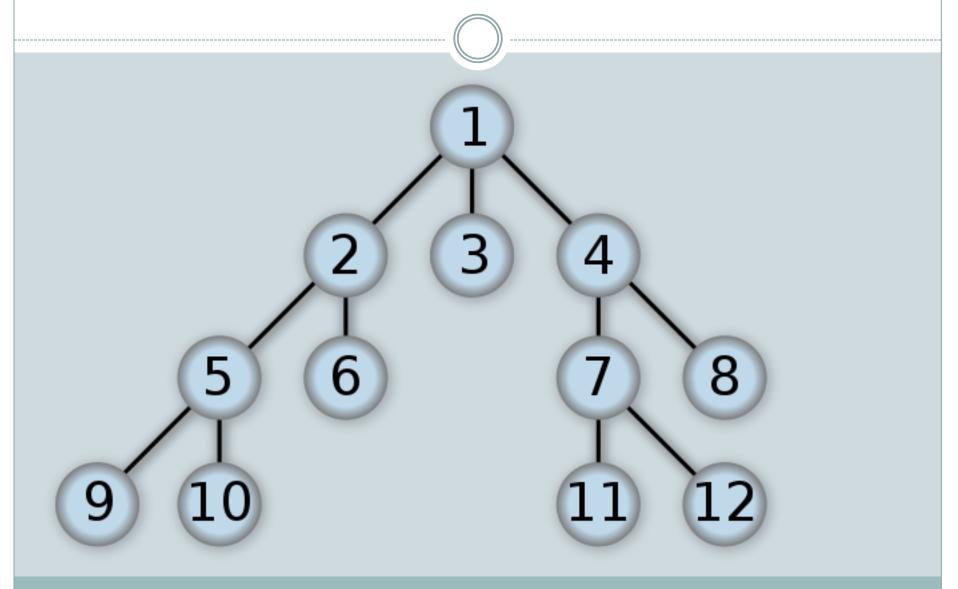
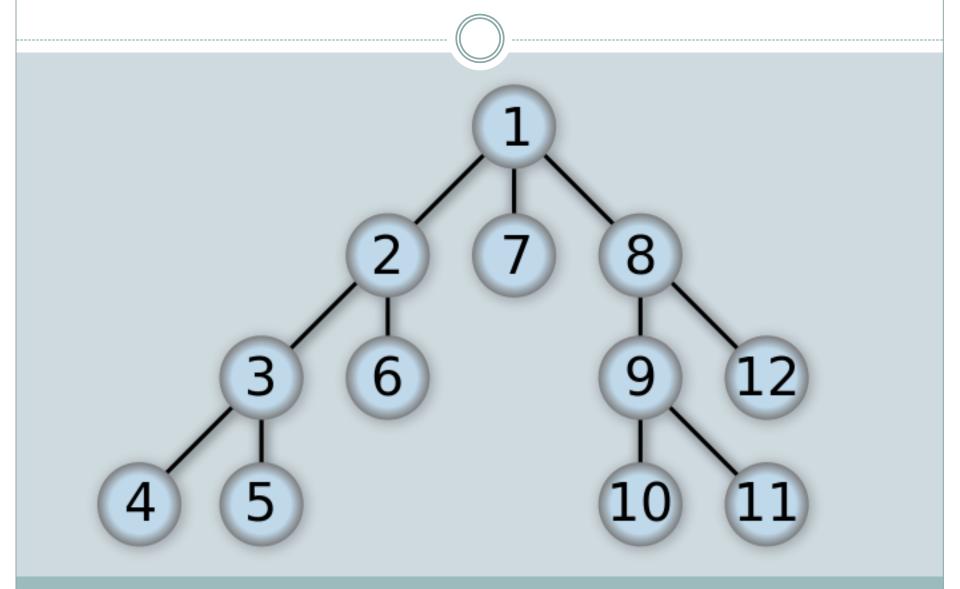
# Heuristic Search

INTRODUCTION TO
ARTIFICIAL INTELLIGENCE
BIL 472
SADI EVREN SEKER

#### Review of Search - BFS



#### Review of Search - DFS



#### Review of Search - IDDFS

- D, F, E, C, G
- DFS Memoryless F, E, A, B, D, F, H
- IDDFS:
  - o Depth o: A
  - O Depth 1: A (repeate
  - o Depth 2: A, B, D, F, C, G, E, F
  - o Depth 3: A, B, D, F, E, C, G, E, F, B

```
• DFS Rememberi IDDFS(root, goal)
                  for(i=1, i<10, i++)
                                                 Е
                    root=i*goal;
```

#### **Blind Search**

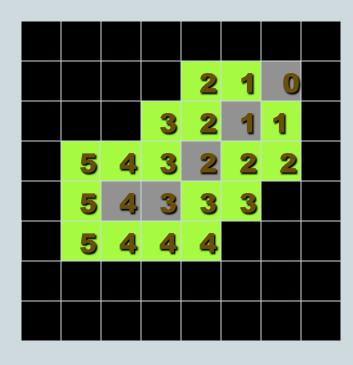
Last time we discussed BFS and DFS and talked a bit about how to choose the right algorithm for a given search problem.

But, if we know about the problem we are solving, we can be even cleverer...

# Revised Template

```
• fringe = \{(s_0, f(s_0))\}; /* initial cost */
markvisited(s<sub>o</sub>);
• While (1) {
 If empty(fringe), return failure;
  (s, c) = removemincost(fringe);
 If G(s) return s;
 Foreach s' in N(s)
    if s' in fringe, reduce cost if f(s') smaller;
    else if unvisited(s') fringe U= {(s', f(s')};
    markvisited(s');
```

#### Cost as True Distance



#### Some Notation

Minimum (true) distance to goal

• t(s)

Estimated cost during search

• f(s)

Steps from start state

• g(s)

Estimated distance to goal (heuristic)

• h(s)

# Compare to Optimal

Recall b is branching factor, d is depth of goal (d=t(s<sub>o</sub>)) Using true distances as costs in the search algorithm (f(s)=t(s)), how long is the path discovered? How many states get visited during search?

# Greedy

True distance would be ideal. Hard to achieve.

What if we use some function h(s) that *approximates* t(s)?

f(s) = h(s): expand closest node first.

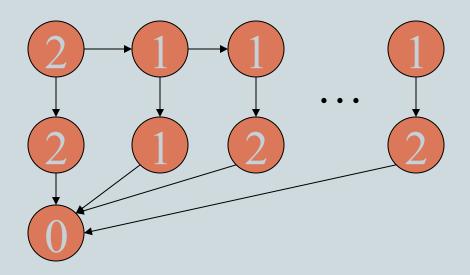
## **Approximate Distances**

We saw already that if the approximation is perfect, the search is perfect.

What if costs are 
$$+/-1$$
 of the true distance?  
 $|h(s)-t(s)| \le 1$ 

# Problem with Greedy

#### Four criteria?



# Algorithm A

#### Discourage wandering off:

$$f(s) = g(s) + h(s)$$

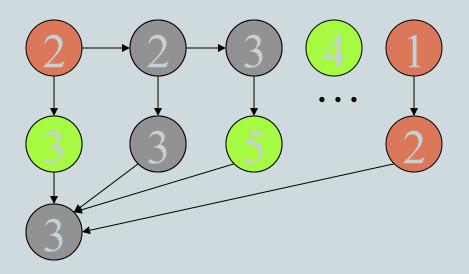
In words:

Estimate of total path cost: cost so far plus estimated completion cost

Search along the most promising path (not node)

#### A Behaves Better

#### Only wanders a little



#### A Theorem

If  $h(s) \le t(s) + k$  (overestimate bound), then the path found by A is no more than k longer than optimal.

#### A Proof

f(goal) is length of found path.

All nodes on optimal path have f(s) = g(s) + h(s)

$$\leq g(s) + t(s) + k$$

= optimal path + k

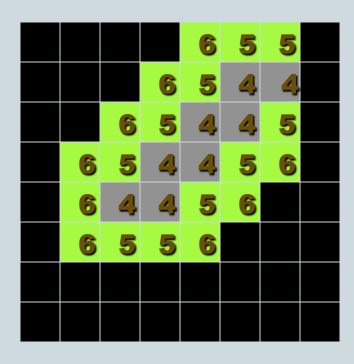
# How Insure Optimality?

Let k=0! That is, heuristic h(s) must always underestimate the distance to the goal (be optimistic).

Such an h(s) is called an "admissible heuristic".

A with an admissible heuristic is known as A\*. (Trumpets sound!)

# A\*Example



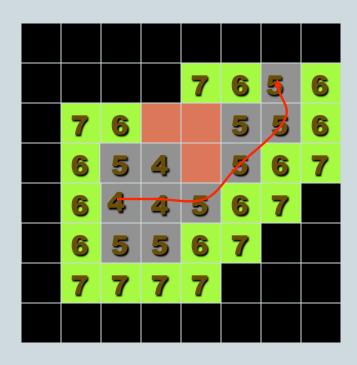
# Time Complexity

Optimally efficient for a given heuristic function: No other complete search algorithm would expand fewer nodes.

Even perfect evaluation function could be O(b<sup>d</sup>), though. When?

Still, more accurate better!

# Simple Maze



# Relaxation in Maze

Move from (x,y) to (x',y') is illegal

- ||x-x'|| > 1
- $\bullet$  Or |y-y| > 1
- Or (x',y') contains a wall

Otherwise, it's legal.

#### Relaxations Admissible

Why does this work?

Any legal path in the full problem is still legal in the relaxation. Therefore, the optimal solution to the relaxation must be no longer than the optimal solution to the full problem.

# Relaxation in 8-puzzle

Tile move from (x,y) to (x',y') is illegal

- If |x-x | > 1 or |y-y'| > 1
   Or (|x-x'| ≠ 0 and |y-y'| ≠ 0)
- Or (x',y') contains a tile

Otherwise, it's legal.

## Two 8-puzzle Heuristics

- h<sub>1</sub>(s): total tiles out of place
- h<sub>2</sub>(s): total Manhattan distance

Note:  $h_1(s) \le h_2(s)$ , so the latter leads to more efficient search

Easy to compute and provides useful guidance

# Knapsack Example

Optimize value, budget: \$10B.

Mark. cost value

• NY 6

• LA 5

• Dallas 3 5

• Atl 3 5

• Bos 3 4

# Knapsack Heuristic

State: Which markets to include, exclude (some undecided).

Heuristic: Consider including markets in order of value/cost. If cost goes over budget, compute value of "fractional" purchase.

Fractional relaxation.

# Memory Bounded

- Just as iterative deepening gives a more memory efficient version of BFS, can define IDA\* as a more memory efficient version of A\*.
- Just use DFS with a cutoff on f values. Repeat with larger cutoff until solution found.

#### What to Learn

The A\* algorithm: its definition and behavior (finds optimal).

How to create admissible heuristics via relaxation.

# Homework 2 (partial)

- 1. Consider the heuristic for Rush Hour of counting the cars blocking the ice cream truck and adding one. (a) Show this is a relaxation by giving conditions for an illegal move and showing what was eliminated. (b) For the board on the next page, show an optimal sequence of boards *en route* to the goal. Label each board with the f value from the heuristic.
- 2. Describe an improved heuristic for Rush Hour. (a) Explain why it is admissible. (b) Is it a relaxation? (c) Label the boards from 1b with the f values from your heuristic.

# Rush Hour Example

